

The vertical structure of ocean eddies

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Abstract

Velocity data from 81 globally distributed current meters are used to characterize the vertical structure of ocean current fluctuations. The primary empirical orthogonal functions from most of the moorings are similar, decreasing monotonically with depth to a value near zero at the bottom. This contrasts with the standard barotropic (BT) and baroclinic (BC) modes, which have flow at the bottom. However, the structure is very similar to the first baroclinic (BC1) mode with zero horizontal flow at the bottom, as is appropriate over a rough or steeply sloping bottom. This mode captures a greater fraction of the observed variance than the standard flat bottom BC1 mode. Also, an analytical approximation of the first mode, obtained assuming exponential stratification, is as accurate as the numerically generated BC mode. This suggests a simple way to project surface velocities into the interior.

Issue Section: [Research article](#)

1. Introduction

Satellite measurements have had an enormous impact on oceanography. Previous to the satellite period, the ocean was

observed primarily from ships, crossing ocean basins in weeks to months, or with modest numbers of acoustically tracked floats. The result was synoptic snapshots of currents in sampled regions. Now, we have $1/4^\circ$ gridded maps of sea surface height (SSH) spanning the global ocean, 1 km maps of sea surface temperature and $1/4^\circ$ maps of sea surface salinity (spatial smoothing applied to the data reduces the actual resolution for these products: roughly 100–200 km for SSH and salinity, and 4–5 km for surface temperature). Thus, we will soon have simultaneous estimates of SSH and density, with global extent.

There remains, however, a long-standing question of how these fields reflect motion below the surface. The issue has been addressed in different ways. [De Mey and Robinson \(1987\)](#) proposed using empirical orthogonal functions (EOFs) to capture the vertical structure. The EOFs, which derive from a singular value decomposition of velocity data at a given location, yield a set of orthogonal basis functions upon which the data can be projected ([Wunsch, 2006](#)). Of particular interest is when the variance is dominated by one or two EOFs, as these provide a simple representation of the vertical structure. Calculating EOFs is straightforward, but to apply this approach globally requires measuring currents everywhere.

However, if one had analytical structure functions, one could potentially deduce the vertical structure from climatological (mean) fields, such as the density. To this end, [Wunsch \(1997\)](#) analysed the vertical structure in terms of *baroclinic (BC) modes*, using data from a large set of current meters. The BC modes comprise an orthogonal basis which can be determined from the density profile, assuming the horizontal velocities are in ‘quasi-geostrophic balance’ ([Gill, 1982](#); [Pedlosky, 1987](#)). Wunsch found that the two gravest modes [the ‘barotropic’ (BT) and ‘first baroclinic’ (BC1) modes] often dominate, accounting for up to 90% of the variance. The dominance of these modes is predicted by theory ([Fu and Flierl, 1980](#)) and has been seen in idealized turbulence simulations ([Scott and Arbic, 2007](#); [Smith and Vallis,](#)

2001). Furthermore, as BC1 has a greater surface signature than the BT mode, Wunsch suggested that SSH primarily reflects BC1. Consistent with this, [Stammer \(1997\)](#) found that SSH anomalies typically have a length scale proportional to that of BC1.

Under the assumption of geostrophic balance, the horizontal surface velocities can be derived from lateral gradients of SSH (Section 2). Wunsch's results suggest one can project these downward using the BC1 vertical structure. However, the BT mode, which is equally important as BC1 at depth, is not well-captured by satellite-derived SSH due to its low temporal resolution (generally 10 days are required for a global survey). Without more information about the BT mode, one could only capture about half the subsurface variance. Wunsch also calculated EOFs at the moorings, but these did not resemble either the BT or BC1 modes but rather a combination of the two. He suggested this was likely the result of the two modes being present in roughly equal proportion.

The BC mode projection applies to SSH but neglects surface density (derived from surface temperature and salinity). Beginning in 2006, several groups explored how density could be projected vertically. The approach they used involves a variant of the quasi-geostrophic construct known as 'surface quasi-geostrophy' (SQG) ([Held *et al.*, 1995](#)). Under SQG, the surface density can be used to develop a 3D pressure field, which in turn can be used to calculate the (geostrophic) horizontal velocities ([LaCasce and Mahadevan, 2006](#); [Lapeyre and Klein, 2006](#); [Isern-Fontanet *et al.*, 2008](#); [Tulloch and Smith, 2006](#)). The SQG-derived velocities were found to be qualitatively similar to *in situ* velocity observations, down to roughly 100 m depth ([LaCasce and Mahadevan, 2006](#)). However, the predicted velocities were consistently too weak, with the amplitude discrepancy increasing with depth.

Alternate approaches were explored. [Scott and Furnival \(2012\)](#) proposed a different set of BC modes, in which the surface pressure was determined solely by the BT mode. They tested combinations

of these modes against a combination of the traditional BT and BC1 modes. [Smith and Vanneste \(2013\)](#) proposed a ‘surface-aware’ set of modes, which have non-zero density at the surface. These modes, however, are non-unique and require additional conditions to close the problem. [Lapeyre \(2009\)](#) examined appending an SQG component onto the BC modes, to account for the surface density. But this is problematic as the SQG solution is not orthogonal to the BC modes ([LaCasce, 2012](#)). Theoretical aspects of these various approaches were discussed recently by [Rocha et al. \(2016\)](#).

Nevertheless, it is possible to combine the SQG and BC modes, if calculated separately ([Lapeyre and Klein, 2006](#)). [Wang et al. \(2013\)](#) showed how the SQG component and the two gravest BC modes could be deduced if one has simultaneous measurements of surface density and surface height. Using data from a numerical ocean model, they were able to reconstruct the flow down to nearly 1000 m. [LaCasce and Wang \(2015\)](#) simplified their approach, by assuming the motion vanishes at depth and by using analytical expressions valid when the density has an exponential profile. This method is appealingly simple, requiring only the e-folding scale of the climatological density. Importantly, the SQG portion was found to be significant only in the upper tens of meters of the water column; below that, the BC mode dominated.

This implies that SSH largely reflects BC1, as suggested by [Wunsch \(1997\)](#). But rather than the traditional BC mode, the correct mode may be one which has *zero horizontal flow at depth*. If true, this would explain the dominant EOF of [Wunsch \(1997\)](#), as both decay monotonically from the surface to near zero at the bottom.

To test this, we analysed data from 81 current meters. These include some of the instruments analysed by [Wunsch \(1997\)](#) and a number of others. Consistently, the dominant EOF closely resembles BC1 with zero flow at the bottom. In addition, the analytical BC1, obtained assuming exponential stratification and zero deep flow, is very similar. This suggests a simple subsurface

projection for SSH.

2. Baroclinic modes

The BC modes derive from the linear quasi-geostrophic potential vorticity (QGPV) equation [5]:

$$\frac{\partial}{\partial t} \left[\nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) \right] + \beta \frac{\partial \psi}{\partial x} = 0, \quad (1)$$

where f_0 is the Coriolis parameter and β is its derivative with latitude, both assumed constant and evaluated at the latitude of interest (the ' β -plane approximation'). Further, $N(z)$ is the buoyancy frequency, defined:

$$N^2 = -\frac{g}{\rho_c} \frac{d\rho_0}{dz},$$

where $\rho_0(z)$ is the background density and ρ_c is a reference density for seawater (roughly 1000 kg/m³).

In addition,

$$\psi = \frac{p}{\rho_c f_0}$$

is the geostrophic streamfunction, which reflects the pressure, p . The surface value is determined from the SSH:

$$\psi(x, y, 0, t) = \frac{g}{f_0} \eta(x, y, t), \quad (2)$$

if η is the surface height (LaCasce and Wang, 2015). The first-order horizontal velocities derive from ψ :

$$(u, v) = \left(-\frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} \right). \quad (3)$$

Assuming a solution which is wave-like in the horizontal:

$$\psi = \sum_{k,l,\omega} \phi(z) e^{ikx+ily-i\omega t}, \quad (4)$$

and substituting this into (1) yields an ordinary differential equation for the vertical structure, $\phi(z)$:

$$\frac{d}{dz} \left(\frac{f_0^2}{N^2} \frac{d\phi}{dz} \right) + \lambda^2 \phi = 0 \quad (5)$$

where $\lambda^2 = -[k^2 + l^2 + \beta/(k\omega)]$. The solutions of (5) depend on the sign of λ^2 . If negative, the solutions decay from the boundaries; these are closely related to the SQG solutions noted previously. If λ^2 is positive, the solutions are oscillatory. With appropriate boundary conditions at the top and bottom, this constitutes a Sturm–Liouville problem, yielding the discrete set of BC modes (Gill, 1982; Kundu et al., 1975; Pedlosky, 1987; Wunsch and Stammer, 1998).

For boundary conditions, one assumes the normal flow vanishes at two surfaces. If the upper surface (at $z = 0$) is taken to be a ‘rigid lid’, the vertical velocity vanishes there. Under QG, this implies the vanishing of the vertical derivative of the streamfunction:

$$\frac{d\phi}{dz} = 0 \quad \text{at } z = 0. \quad (6)$$

The rigid lid assumption is made for simplicity; of course, if the sea surface were actually rigid, the BC modes would not be visible from space. In reality, there is a surface deviation, and this is anti-correlated with the deviation of the subsurface ‘thermocline’ [the maximum gradient in the subsurface density (Wunsch and Gill, 1976)]. A free surface condition can be applied instead (Gill, 1982;

Wunsch and Stammer, 1998), but this does not change the modal structure qualitatively. Condition (6) also implies the perturbation density vanishes at the surface, which follows from the hydrostatic relation:

$$\frac{\partial p}{\partial z} = -\rho g = \rho_c f_0 \frac{\partial \psi}{\partial z} = \rho_c f_0 \sum_{k,l,\omega} \frac{d\phi}{dz} e^{ikx+ily-i\omega t}. \quad (7)$$

So, the BC modes have no signature in surface density.

The fluid depth is taken to be $H = H_0 - h(x, y)$. Under QG, the deviation, h , is assumed to be much less than the mean depth, H_0 . The no-normal flow condition at the bottom implies:

$$\vec{u}(-H) \cdot \nabla h = 0 \quad (8)$$

The standard choice is to take the bottom to be flat ($h = 0$). Then, w , and hence $d\phi/dz$, vanish at $z = -H$. If bottom topography is present, the vertical velocity does not vanish. However, if the topography is either steep or rough, the no-flow condition can be satisfied if the horizontal velocity, $\vec{u}(-H)$, vanishes at the bottom. Using this condition also yields a set of BC modes, but with no flow at the bottom (Bobrovich and Reznik, 1999; Charney and Flierl, 1981; Rhines, 1970; Samelson, 1992; Straub, 1994; Tailleux and McWilliams, 2001; Veronis, 1981). Support for the horizontal flow vanishing at depth has been found both in numerical studies (Arbic and Flierl, 2004; Boning, 1989; Killworth, 1975; LaCasce, 1998, 2012; LaCasce and Brink, 2000; McWilliams *et al.*, 1986; Owens and Bretherton, 1978; Rhines, 1981; Treguier and Hua, 1988) and in observations (Dickson, 1983; Wunsch, 1983).

Thus, we have:

$$\frac{d\phi}{dz} = 0 \text{ or } \phi = 0 \text{ at } z = -H. \quad (9)$$

Either condition yields a complete set of orthogonal eigenmodes. We refer to the Neumann case as the ‘flat bottom’ one and the Dirichlet case as the ‘rough bottom’ one (recognizing that the condition also applies in the case of a smooth but steep bottom slope). We solve (5–9) numerically at each mooring location.

The modes can also be obtained analytically for certain profiles of the buoyancy frequency, N . One such profile is an exponential. Assuming $N = N_0 e^{\alpha z}$, the BC modes which satisfy the rigid lid condition at $z = 0$ have the form (LaCasce, 2012):

$$\phi = A e^{\alpha z} [Y_0(\gamma) J_1(\gamma e^{\alpha z}) - J_0(\gamma) Y_1(\gamma e^{\alpha z})] \quad (10)$$

where $\gamma = N_0 \lambda (\alpha f_0)^{-1}$, A is a constant and the J ’s and Y ’s are first- and second-order Bessel functions (LaCasce, 2012 assumed instead that $N^2 = N_0^2 e^{\alpha z}$, so α here corresponds to $\alpha/2$ in his expressions). Applying either the Neumann or Dirichlet condition at the bottom yields a transcendental equation for γ which can be solved using Newton’s method.

There is an important distinction between the modes obtained with the different bottom conditions. With a flat bottom, the eigenmodes include the depth-independent BT mode. But having zero flow at the bottom precludes the BT mode. In reality, the BT mode is replaced by a bottom-intensified topographic wave (Charney and Flierl, 1981; Rhines, 1970), but this is not obtained in the Sturm–Liouville solution for the BC modes. So, while the rough bottom modes are formally complete, they have no horizontal velocity at the bottom. This poses a problem when reconstructing a vertical profile that actually has bottom flow. Ideally, one would subtract the topographic wave contribution from a given profile prior to projecting onto the rough bottom BC modes. But to do so requires knowledge of the topographic waves, whose vertical extent depends on their horizontal wavelength. As such, one cannot simply project a given profile onto the rough bottom modes.

One can take a practical approach though. As noted, SSH is usually taken to reflect BC1 over a flat bottom. We will demonstrate that if one assumes instead that it reflects the rough bottom BC1, one can capture a greater fraction of the subsurface variance.

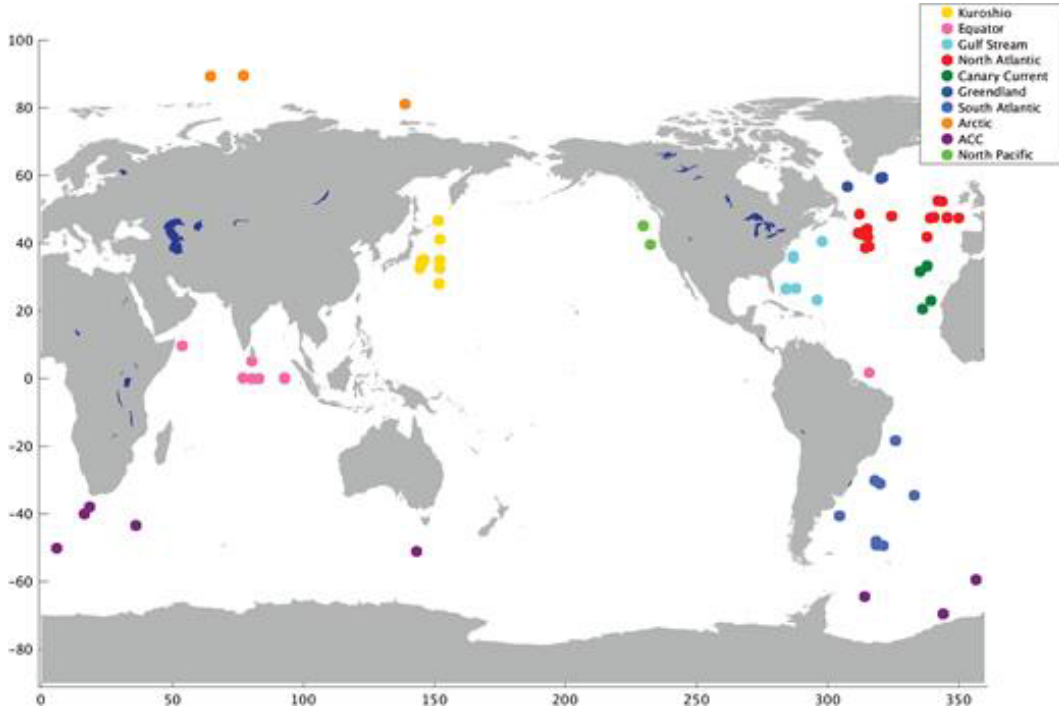
3. Data and methods

We employ a set of current meter records from the Global Multi-Archive Current Meter Database (GMA-CMD) compiled by R. B. Scott (<http://stockage.univ-brest.fr/scott/GMACMD/updates.html>). The records include those studied by Wunsch (1997) and many additional ones. We selected moorings whose instruments span a large fraction of the water column and whose records are relatively long. In particular, we required that the moorings:

1. have at least four instruments,
2. have instruments which span at least 75% of the water column and
3. have records which are at least 300 days long (allowing gaps).

Doing this, we obtained 81 moorings (out of the 3258 moorings in the database), at the locations shown in Figure 1. Many moorings are in the North Atlantic, but there are also examples in the North Pacific, South Atlantic, North Indian, Southern and Arctic Oceans. We group these into clusters according to location. We filtered the time series with a Butterworth filter to obtain daily velocities in all cases.

Figure 1.



Positions of the 81 moorings that meet the specified requirements. The moorings are coloured according to location.

For the density profiles, we used climatological temperature and salinity data from the World Ocean Atlas 2000 (WOA09). The profiles represent annual means, so seasonal variations are neglected. The potential density is calculated following [Gill \(1982\)](#), using the Matlab routine `sw_pdens.m`.

The BC modes will also be compared to EOFs. The latter are obtained by constructing a covariance matrix $R = U'U$ from the observations, where each column of U holds a velocity time series from a different depth at the mooring. Then, one solves the eigenvalue problem ([Von Storch and Zwiers, 2001](#)):

$$RC = C\Lambda . \quad (11)$$

We do this for the u and v components independently, and use standard (non-weighted) EOFs. The diagonal matrix, Λ , contains the eigenvalues χ_i of the covariance matrix that correspond to each EOF eigenvector. Each mode is classified by the percentage of

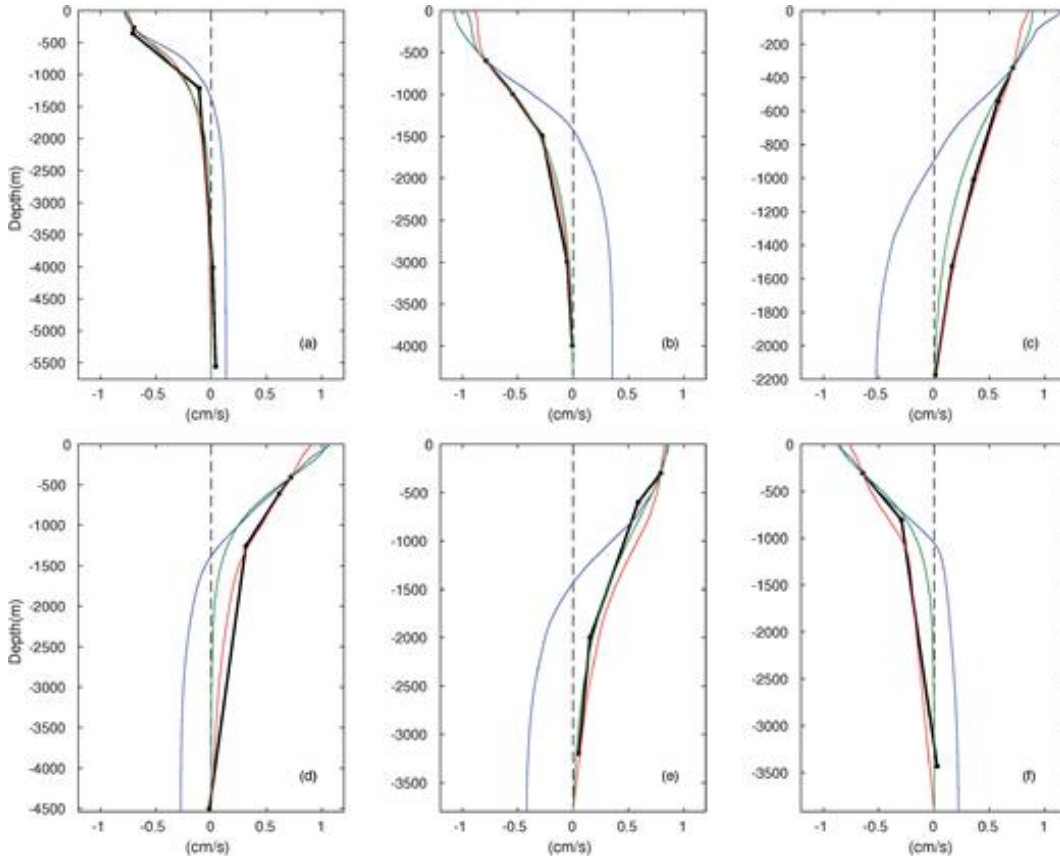
variance (PEV) explained, i.e. $PEV_i = \chi_i / (\sum_i \chi_i)$, and the EOFs are ordered accordingly, with the first mode having the largest PEV.

The EOFs can be constructed in such a way as to satisfy the surface and bottom boundary conditions. We chose not to do this, as it might bias the structures in favour of the flat or rough bottom solutions. Rather, we leave the boundary conditions unspecified and compare the resulting structures with the respective BC modes.

4. Results

We begin with several representative examples. Shown in [Figure 2](#) are the first EOFs from six moorings, from the Kuroshio region (a), the North Atlantic (b), the South Atlantic (c), the Canary Current (d) the ACC (e) and the Gulf Stream (f). Consider the North Atlantic case (b). EOF1 (the black curve with dots), which accounts for 83% of the variance, decays smoothly from the surface to near zero at the bottom with a vertical scale of roughly 750 m. The flat and rough bottom BC1s are also shown, with amplitudes chosen to match that of EOF1 at the shallowest instrument. The flat bottom BC1 (in blue) has a similar vertical scale, but it crosses zero near 1250 m and has oppositely signed flow at the bottom. The rough bottom BC1 (in red) does not cross zero and closely resembles EOF1.

Figure 2.



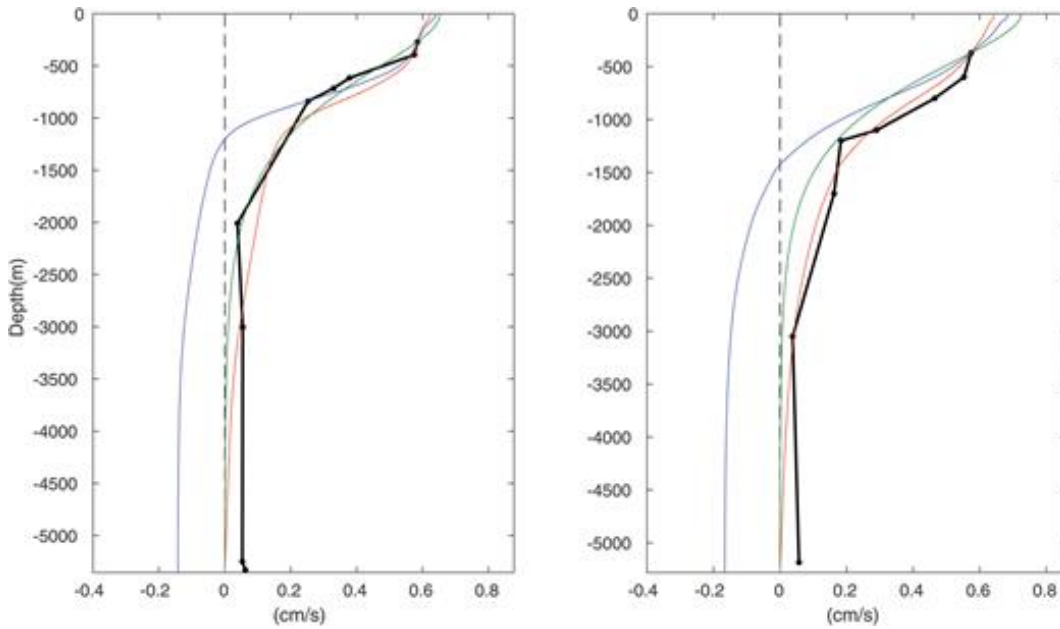
Principal EOFs from six representative moorings, from (a) the Kuroshio, ($PEV_1 = 0.93$), (b) the North Atlantic ($PEV_1 = 0.83$), (c) the South Atlantic ($PEV_1 = 0.78$), (d) the Canary Current ($PEV_1 = 0.96$), (e) the ACC ($PEV_1 = 0.95$) and (f) Gulf Stream ($PEV_1 = 0.86$). Shown for comparison are the flat bottom BC1 (in blue), the rough bottom BC1 (in red) and the analytical rough bottom BC1 assuming exponential stratification (in green).

The results are similar in the other cases, with EOF1 decaying monotonically with depth towards zero at the bottom. The rough bottom BC1 behaves the same and has a comparable vertical decay scale, while the flat bottom BC1 crosses zero and has opposed flow at the bottom. Note that in cases (a), (d) and (f), the EOFs do actually cross zero. But this occurs at great depth (below 3000 m), well below the zero-crossing depth of the flat bottom BC1, and where the velocities are very small.

Also shown in Figure 2 are the analytical BC1 curves derived assuming exponential stratification (in green). These also closely resemble the first EOFs. We return to this solution in Section 4.2.

The moorings in Figure 2 have only four to five instruments, and one might wonder whether the observed structure is a result of having too few observations with depth. To check this, we relaxed the record length requirement to 100 days and obtained 10 moorings with at least eight instruments in the vertical. Six of these are located near the equator (where the results differ somewhat, as discussed below). Two, however, are in the extratropics, in the Gulf Stream and Canary Current regions. The primary EOFs from these moorings (Fig. 3) also decay monotonically with depth to small values (<1 mm/second) at depth, without crossing zero. And again, the structure is fairly well-captured by the rough bottom BC1.

Figure 3.

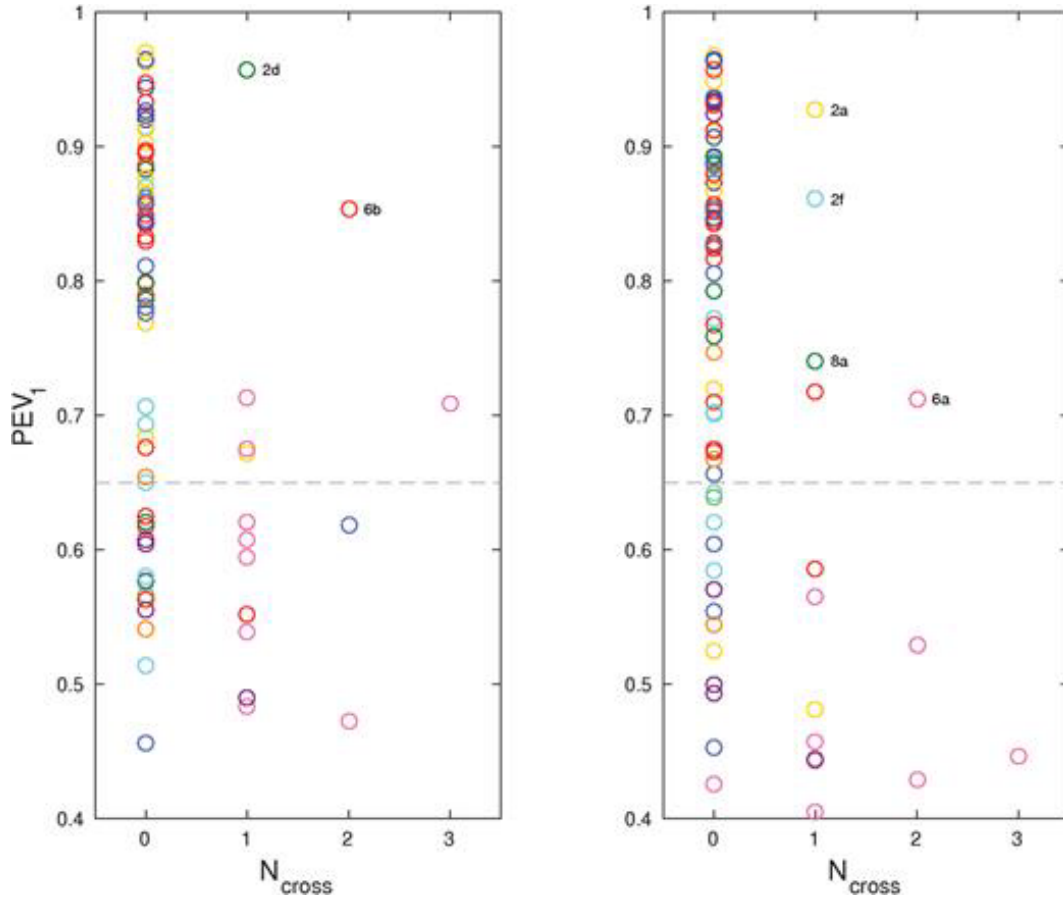


As in Figure 2 but for two cases with a large (eight) number of instruments in the vertical. The moorings are from the Gulf Stream (left) and Canary Current (right) regions, with $PEV_1 = 0.81$ and $PEV_1 = 0.69$, respectively.

The first EOFs for many of the other moorings are similar. One way to see this is simply to check the number of zero crossings of EOF1. This is shown in Figure 4, plotted as a function of the PEV captured by the first mode. The results for u are shown in the left panel and for v in the right panel, and the colour coding indicates where the

moorings are located, as in [Figure 1](#).

Figure 4.



Number of zero crossings and percentage of explained variance corresponding to the dominant EOF (PEV_1). The left (right) panel corresponds to the u (v) component of the velocity field. The dashed lines indicate the cut-off $PEV_1 = 0.65$, above which the EOFs have $PEV_1/PEV_2 \geq 2$. The labels indicate the figures where particular cases can be seen and the colour coding is as in [Figure 1](#), indicating mooring location.

In the majority of cases, EOF1 has no zero crossings. This is particularly true of those where EOF1 accounts for most of the variance. When EOF1 captures $>65\%$ of the variance in these cases, the first eigenvalue is more than twice the second. We use this as a criterion to distinguish when the variance is essentially captured by a single vertical mode. Among such cases, there are six in which EOF1 has one or more zero crossings for u , and five for v . Three of these (labelled 2a, 2d and 2f in the figure) are the cases with deep

zero crossings noted in [Figure 2](#). Other examples are considered hereafter and are labelled in correspondence to the figures. Many with one or more zero crossings are coloured pink, indicating they are in the tropics. Nevertheless, the majority have no zero crossings.

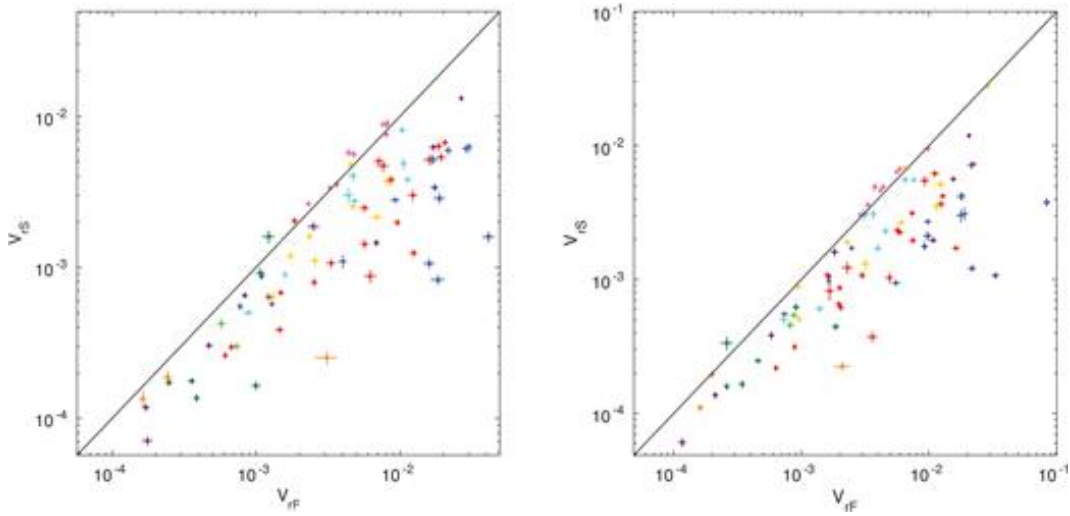
As noted, the lack of the topographic wave mode hinders projecting the observed velocities onto the rough bottom modes. But we can compare using the flat and rough bottom BC1s by determining which captures a greater fraction of the subsurface variance. To do this, we calculated the mean square difference of the velocities at a given mooring from those for the respective modes, i.e.:

$$V_r \equiv \frac{1}{M-1} \sum_{i=1}^M < (v'(z_i) - \zeta(z_i))^2 > , \quad (12)$$

where ζ is either of the two BC1s, M is the total number of instruments on the mooring, each located at z_i , and the brackets imply averaging in time. We set the amplitudes of the BC modes by demanding that each match the velocity at the shallowest instrument (removing 1 d.f., hence the division by $M - 1$). Thus, the greater V_r , the larger the deviation for the deeper instruments. Note this comparison does not employ EOFs—it simply tests the differences between the observations and the prescribed modes.

The results are shown in [Figure 5](#). The residual variances for the rough bottom BC1 are on the y-axis and for the flat bottom BC1 on the x-axis, for the zonal (left) and meridional (right) velocities. Most of the points fall below the line, indicating the variances with the rough bottom BC1 are less. [Figure 5](#) is a log–log plot, so the differences are often a factor or 2 or greater. Very few lie at or above the line, and many of these are coloured pink, indicating equatorial moorings (primarily in the Indian Ocean). A few others are green (Canary Current) or orange (Arctic Ocean). Several of these exhibit one or more zero crossings in EOF1 ([Fig. 4](#)) and are examined hereafter.

Figure 5.



The residual variance of observed currents against the rough bottom BC1 (y-axis) and the flat bottom BC1 (x-axis), for u (left) and v (right) components. The crosses indicate the uncertainties in both variances. The colour coding is as in [Figure 1](#), indicating where the moorings are located.

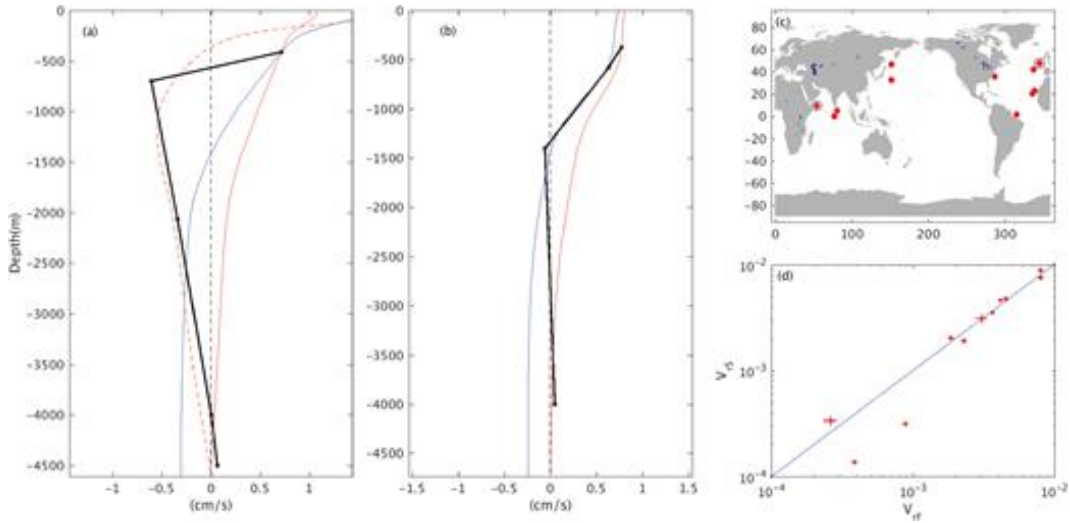
4.1. Exceptions

Thus, the velocities at many of the moorings exhibit similar vertical structure, but there are also exceptions. In some cases, EOF1 exhibits one or more zero crossings and, in others, the EOF decays differently than predicted by the BC1 mode. We consider these in turn. Hereafter, we focus on the cases where the first EOF accounts for $>65\%$ of the variance, i.e. those where the variance is better captured by a single mode.

4.1.1. Zero crossings

Two examples of cases with zero crossings are shown in [Figure 6](#). One (left panel) is from a mooring in the equatorial Indian Ocean, off the horn of Africa, and one (right panel) is from the north-east Atlantic, south of the UK. The positions are indicated by the two circled red dots in the map insert.

Figure 6.



Two first EOFs which exhibit zero crossings, from the meridional velocity at a mooring in the equatorial Indian Ocean (left panel, $PEV_1 = 0.71$) and from the zonal velocity from one in the north-east Atlantic (right, $PEV_1 = 0.85$). The flat bottom (blue) and rough bottom (red) BC1s are shown for comparison. The rough bottom BC2 is also shown in the left panel (dashed red). The locations of similar moorings are indicated in the map insert at upper right, and the two cases plotted are indicated by circled red dots. The residual variances against the flat and rough bottom BC1s are shown in the insert lower right.

For the equatorial mooring, EOF1 crosses zero at around 700 m and again near 4000 m. The second crossing occurs at great depth where the velocities are weak, but the change in sign at 700 m is clear. As such, the mode resembles neither the flat nor the rough bottom BC1. However, it is similar to the *second* rough bottom mode, indicated by the dashed curve. This crosses zero at a somewhat shallower depth, but decays to zero in a similar fashion.

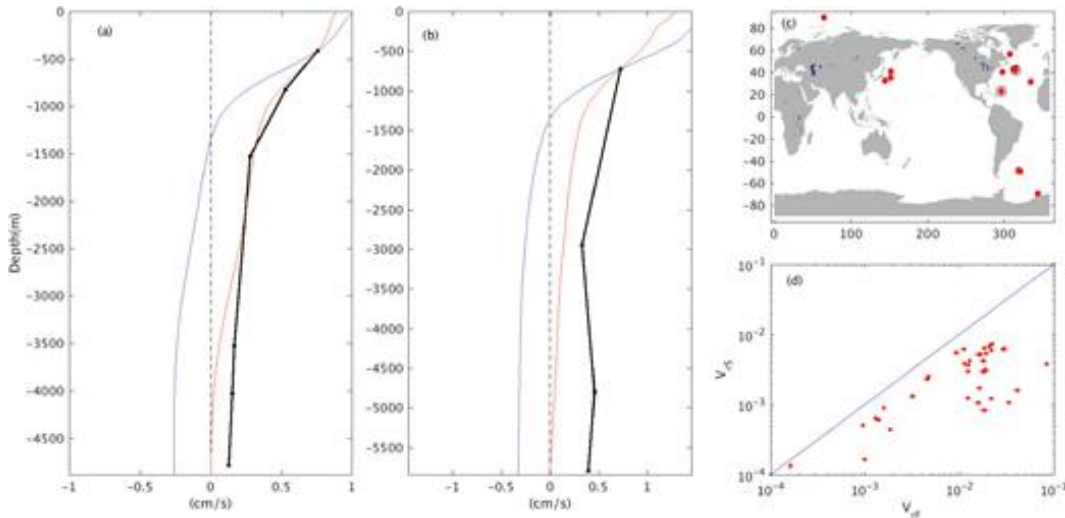
The zero crossing for the eastern Atlantic mooring occurs near 1400 m. This is near the zero crossing of the flat bottom BC1, but the EOF has weak flow at the bottom rather than opposed flow. So, it is possible instead that this mode incorrectly captures the decay to zero, having only two instruments below 700 m. The first EOF for the meridional velocity does not exhibit a zero crossing at all (not shown), suggesting that that seen here may be anomalous.

The other examples (indicated in the inserted map) are similar. With two exceptions, the residual variance is comparable for the rough and flat bottom BC1s (insert, lower right), so neither mode is preferred. But the issue with these cases may not be the choice of bottom boundary condition, but rather the assumed dominance of BC1.

4.1.2. Faster/slower decay towards zero

In other cases EOF1 does not cross zero but decays with depth either faster or slower than expected for the BC modes. The cases in Figure 7 are of the latter type. EOF1 is largest near the surface, but the flow is non-zero at the deepest instruments. Many of the relevant moorings are found at higher latitudes, with two being in the Arctic and another in the Southern Ocean.

Figure 7.



Examples in which EOF1 decays slower than the BC1 modes. The left panel is for v from a mooring in the Gulf Stream extension and the right is for v from one in the Caribbean. The locations of all similar moorings are shown in the insert in the upper right. The squared mean deviations from the rough and flat bottom BC1 modes are in the insert lower right. For the EOFs depicted here, $PEV_1 = 0.84$ (left) and $PEV_1 = 0.70$ (right).

The case in the left panel is for the meridional velocity from a mooring in the North Atlantic, in the Gulf Stream extension. The

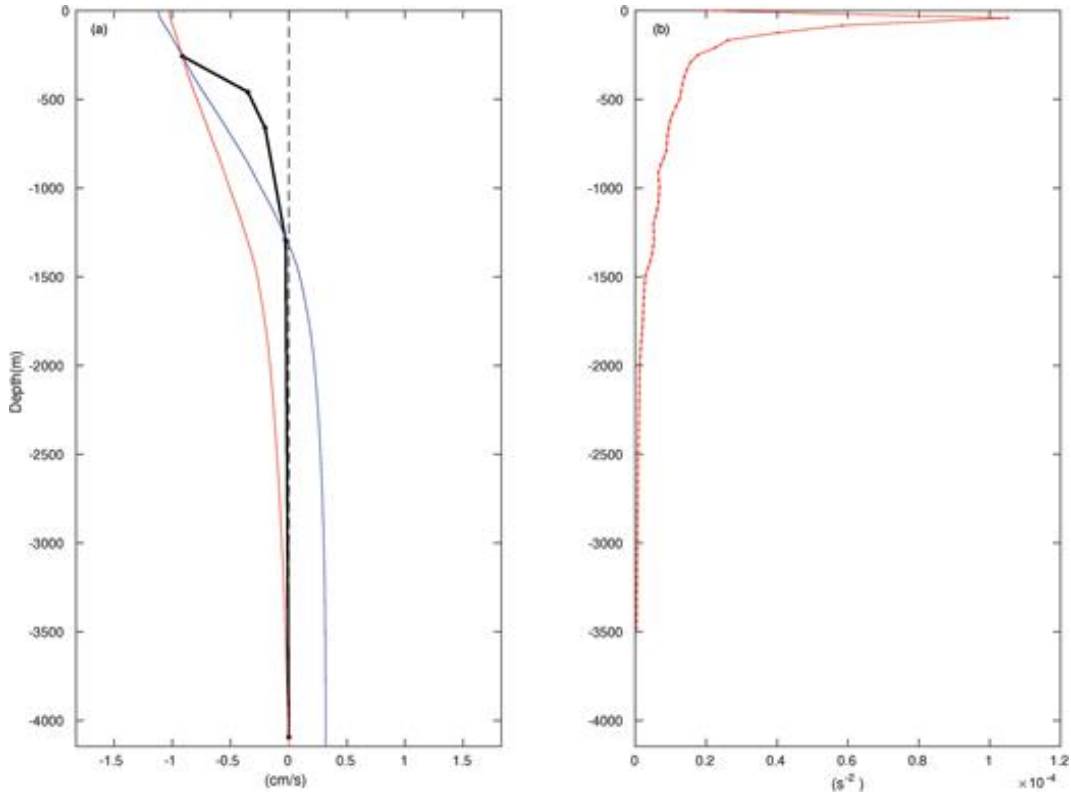
first EOF overlies the rough bottom BC1 from 400 m down to roughly 2000 m, but decays more slowly below that. The EOF in the right panel is from the Caribbean and is nearly depth-independent.

The Caribbean case is suggestive of the BT mode. But most of the cases exhibit surface intensification, like the Gulf Stream example, as expected for the BC modes. And because these EOFs do not cross zero, they most closely resemble the rough bottom BC1. Thus, the squared deviation from the rough BC1 is much less than from the flat BC1 (lower right panel). Hence, the rough bottom mode is still the better choice, without additional information.

The stratification in these cases was usually weak at depth, and this undoubtedly influences the slower decay of EOF1 there. At the same time, the stratification was found to be intensified in the upper 500 m where many of the moorings lack instruments. Had there been more complete vertical sampling, it is possible the first EOF would have been more sheared.

In other cases, EOF1 decays faster than expected. This is very rare, occurring at only two moorings, both located near 40°N. One is shown in [Figure 8](#). The EOF (left panel) decays rapidly, primarily in the upper 500 m. The BC modes decay more gradually, over a scale roughly twice as great.

Figure 8.



A case with a too-rapid decay in the vertical, from the meridional velocity at a mooring at 23°N, near the Canary Current. In the left panel, the dominant EOF ($PEV_1 = 0.74$) is in black, while the flat and rough bottom BC1s are depicted in blue and red, respectively. In the right panel is the climatological stratification at the mooring location.

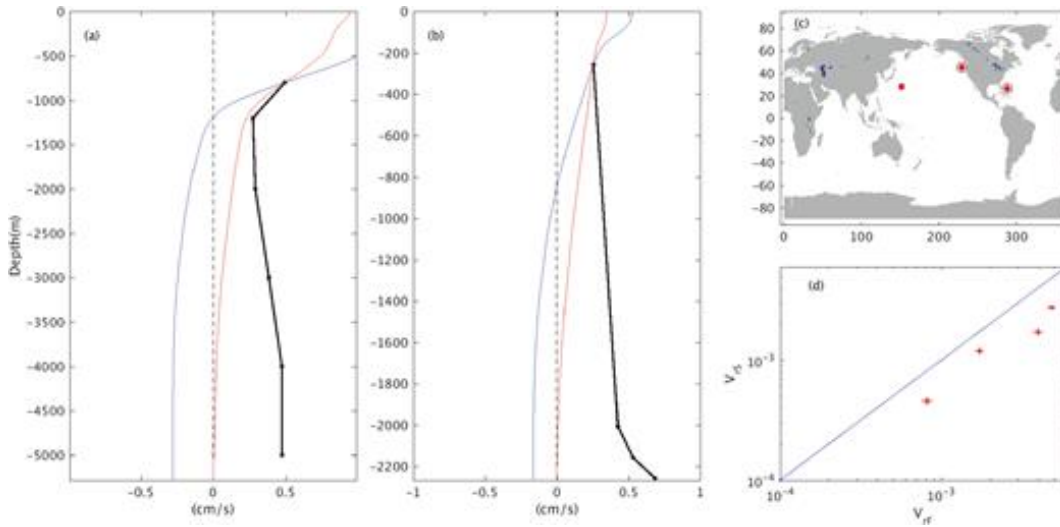
As noted, the BC mode amplitudes match the shallowest instrument, which lies near 300 m. The stratification on the other hand is greatest in the upper 100 m (right panel). So, this could also be a case in which the EOF incorrectly captures the variation with depth due to a lack of sampling in the highly stratified surface region. But as with the too-slow decay cases, the residual variances indicate these EOFs are closer to the rough bottom BC1 than the flat bottom one.

4.1.3. Bottom-intensified fluctuations

At four of the moorings, EOF1 exhibits bottom intensification (Fig. 9). The cases shown are from moorings off the north-west USA (left panel) and near Florida (right panel). The former EOF exhibits

surface intensification but then increases in amplitude below 2000 m. The second example has only a single instrument in the shallow depths (at 250 m), but also exhibits clear bottom intensification below 2000 m.

Figure 9.



EOF1s that exhibit bottom intensification, both with $PEV_1 = 0.70$. The example at left is from the north-east Pacific and the one on the right is from the Caribbean, and both involve the meridional velocity. All moorings with similar behaviour are indicated in the map at right. The panel at bottom right again shows the residual variance between with the flat and rough bottom BC1s.

Neither the flat nor rough bottom BC1 can account for bottom-intensified motion, but such dependence is consistent with topographic waves. Indeed, if the rough bottom modes apply, one would anticipate cases in which the topographic mode is actually dominant. The vertical scale of a topographic wave is proportional to N/f_0 times its horizontal wavelength, so the vertical decay seen here could be used to infer the size of the wave. Indeed, if the waves are large enough, they would produce nearly BT motion. Thus, this case is not necessarily inconsistent with the concept of rough slope modes.

Of course, the surface intensification seen in the left panel of [Figure 9](#) is not possible with topographic waves. It might be the

EOFs reflect a mixed contribution of a topographic wave and rough bottom BC1. As noted, topographic waves are not orthogonal to the rough bottom BC modes, so both could conceivably project onto the gravest EOF. The first EOFs in any case do not resemble a flat bottom BC mode, and the residual variance is greater against that mode than the rough bottom BC1 (insert lower right).

Thus, the exceptions still favour the rough or sloping bottom boundary condition over the flat bottom one when attempting to capture the variance with a single mode. In some cases, BC1 decays differently with depth than expected, but this could be due to under-sampling with the current meter. In other cases, the dominant EOF exhibits bottom intensification, like a topographic wave, but this could also reflect a non-flat bottom.

4.2. Analytical solution

The vertical structure is also reasonably well-captured using the analytical BC1 mode assuming exponential stratification, given in (10). The green curves in [Figure 2](#) were obtained using these modes. In most cases, the curves resemble both the numerically derived rough bottom modes and the primary EOF.

A major advantage of using the exponential solution comes with fitting the stratification. The Brunt–Vaisala frequency is often noisy, which hinders the numerical solution of the Sturm–Liouville problem for the modes. But if N can be fit with an exponential, so can the density:

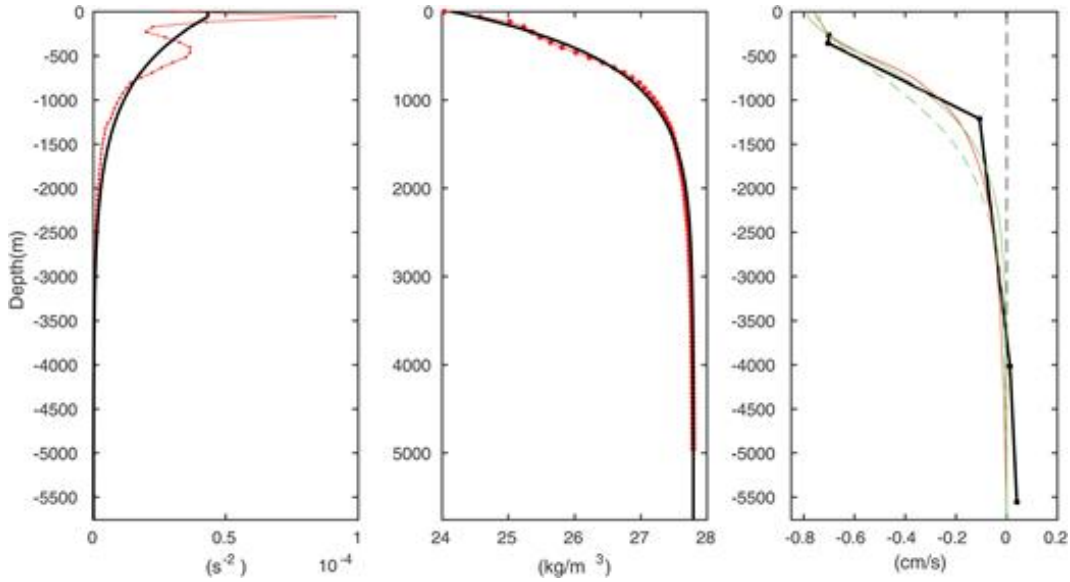
$$N^2 \propto \frac{d\rho}{dz} \propto e^{2\alpha z} \rightarrow \rho \propto e^{2\alpha z} \quad (13)$$

As the density is the integral of N^2 , it is smoother and easier to fit.

An example, from one of the cases in [Figure 2](#), is shown in [Figure 10](#). Fitting $N^2 \propto e^{2\alpha z}$, we obtain $\alpha_N^{-1} = 718m$ with a regression coefficient $R^2 = 0.88$. Fitting the density $\rho \propto e^{2\alpha z}$, we find instead

$\alpha_\rho^{-1} = 578m$ and $R^2 = 0.99$. In the first case, we exclude the depths in the mixed layer $z = [0, z(N_{max})]$; while in the second case, we fit the entire profile. In the right panel are the corresponding two rough bottom BC1 profiles, compared to EOF1 for Figure 2a. The solution using the density fit (solid green) is nearer EOF1 than that obtained fitting N^2 (dashed green).

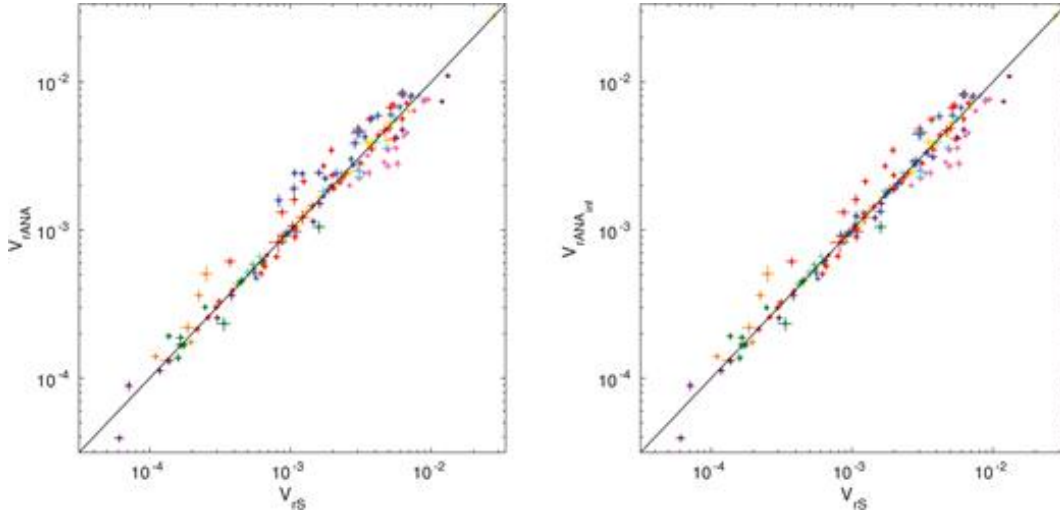
Figure 10.



Stratification (left) and potential density (centre) at the mooring shown in Figure 2a, with exponential fits. In the right panel are EOF_1 and the rough bottom BC1s for both fits.

We calculated the residual variances using the analytical rough bottom BC1 (10) and compared those to the variances obtained using the numerical rough bottom BC1 (left panel of Fig. 11). The variances are very comparable, with some exceptions where the analytical mode captures either more or less of the variance. As might be expected, these are usually cases where the stratification differs more from an exponential, so that the vertical decay, often at depth, differs. But in the majority of the cases the analytical solution is as good as the numerical one.

Figure 11.



The residual variances using the analytical rough bottom solution compared to the numerical flat bottom solution (left panel), and compared to the numerical rough bottom solution (right panel).

We also tested an alternate form of the exponential solution, used by [LaCasce and Wang \(2015\)](#). This vanishes with depth, rather than at the ocean bottom, $-H$. The solution is:

$$\phi(z) = Ae^{\alpha z} J_1(2.4048e^{\alpha z}). \quad (14)$$

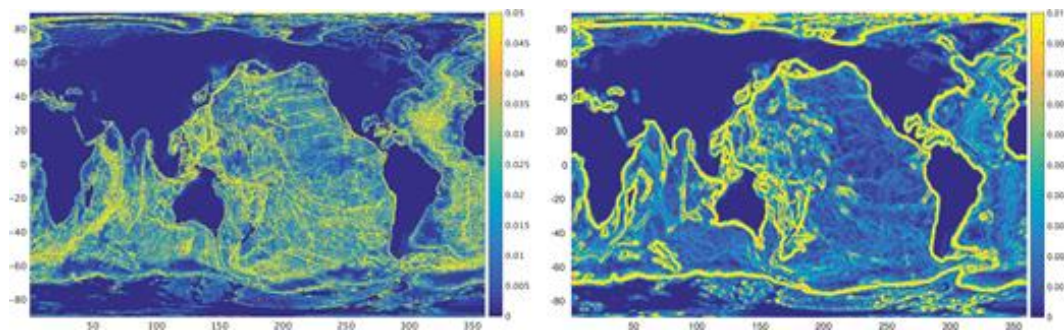
This advantageously does not require solving a transcendental equation to obtain the eigenvalue. One need only fit the density to an exponential and the solution follows. The variances using (14) are very similar to those using (10) (right panel of [Fig. 11](#)). In fact, there is actually improvement with some of the outliers using this solution. The reason is that this solution does not go to zero at the bottom when the water depth is not much greater than the e-folding scale of the stratification. Thus, when EOF1 also decays too slowly, the variance is less. But in many cases this solution produces the same results as with (10), as would be expected when the water depth greatly exceeds the e-folding scale of the stratification.

4.3. Bottom dependence

The vertical structure does not vary greatly geographically, with the exception of the very low and high latitudes. But the question remains of why the dominant mode has weak bottom flow; is it due to bottom roughness, the bottom slope or friction?

To address this, we categorized the moorings by bottom slope and roughness. We calculated the bottom slope, first from the raw eTopo1 data set (with 1 minute resolution), and then from the same set smoothed at 1° resolution. The first yields an indication of bottom roughness (left panel of Fig. 12), while the second highlights strong slope regions, like the continental margins (right panel of Fig. 12). We then plotted the ratio of the variances from the flat and rough bottom BC1s against the topographic gradient, using both the rough and smoothed bottom slopes.

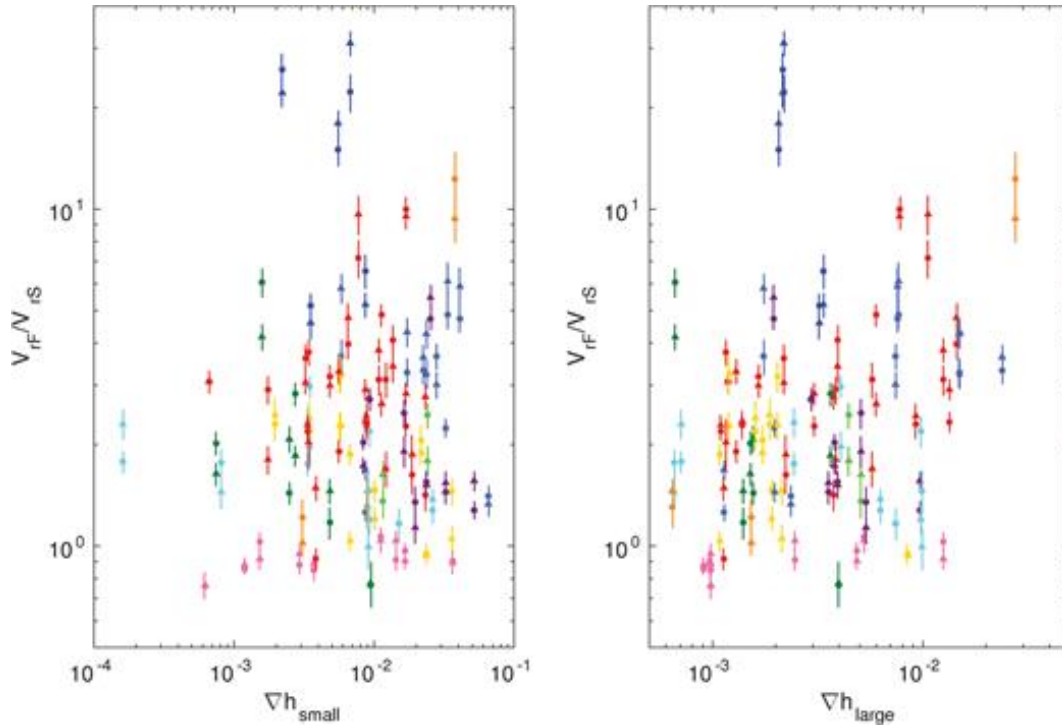
Figure 12.



Topographic slope from eTopo1, calculated at the product resolution (0.0167° , left) and smoothed to 1° resolution.

These are shown in Figure 13. In nearly all cases, the ratio is >1 , indicating a closer fit with the rough bottom BC1. However, the ratio does not vary consistently with either the rough or smoothed bottom slope. So, the tendency for weak bottom flow is the same in all regions, regardless of bottom roughness or slope.

Figure 13.



The ratio of the residual variances for the flat and rough bottom BC1 versus topographic slope at each mooring location, with the raw eTopo1 data set (left panel) and from the smoothed 1° topography (right).

This points towards bottom friction or the combined action of bottom friction and topography as potential causes for the shutdown of the deep flow. As noted previously, there are many indications of weak bottom flow in numerical ocean simulations. As seen in 2D turbulence simulations ([Arbic and Flierl, 2004](#); [Arbic and Scott, 2008](#); [LaCasce and Brink, 2000](#)), friction and bottom topography act in similar ways to decouple the flow at depth, and bottom friction alone can also be effective in this regard.

We also examined whether the vertical structure of EOF1 varies on different time scales, by using low- and high-passed time series with a range of cut-off frequencies. One might expect, for example, that high frequency motion is more BT and the low frequency fluctuations are more BC ([Willebrand et al., 1980](#)). However, no consistent variation was seen. In a few cases, the EOF1 obtained with high frequency velocities appeared more BT; while in others, there was no variation at all with frequency. It

seems the surface intensification is a generic part of the response.

5. Summary and discussion

We have examined the vertical structure of fluctuating currents using a set of 81 current meter moorings. The primary EOFs calculated for the moorings are frequently similar, decreasing monotonically with depth to near zero at the bottom, as noted earlier by Wunsch (1997). We demonstrate that this structure resembles the BC1 mode obtained with zero horizontal flow at the bottom (rather than zero vertical velocity, as assumed with traditional ‘flat bottom’ BC modes). Such a ‘rough bottom’ BC mode also resembles a combination of the BT and BC1 modes with a flat bottom, under the condition that the two cancel each other at the bottom. In the present conception, the observed structure is explained with a single mode rather than a combination of two.

One disadvantage of the rough bottom modes is that one cannot simply project a given vertical profile onto them, because the topographic wave mode (which substitutes for the BT mode) is excluded by the bottom boundary condition. In doing such a projection, one would ideally subtract the topographic wave contribution—at each time—before making the projection onto the BC modes. But this requires knowing the size of the wave, which cannot be resolved with a single current meter. Nevertheless, the observed vertical structure is closer to that of the rough bottom BC1 than the flat bottom BC1, so it is preferable to assume the rough bottom BC1 when interpreting the subsurface structure associated with SSH.

An analytical version of the rough bottom mode, given in (10) and obtained assuming exponential stratification, is often as accurate as the numerically derived rough bottom mode. Having this solution makes the vertical projection very straightforward. Given the climatological stratification at a given location, one fits the

density profile with an exponential to obtain the e-folding depth, α . One then solves the transcendental equation which results from evaluating (10) at $z = -H$ and setting it equal to zero. The vertical structure is given by (10), using the first value of γ obtained from the transcendental equation. In deep water applications, the approach is even simpler because the structure is well-captured by a solution (14) which simply vanishes with depth. This avoids solving a transcendental equation; one only requires the e-folding scale of the stratification.

The dominant mode, however, is not always surface-intensified; occasionally it also exhibits bottom intensification, suggestive of a topographic wave contribution. One might expect strong topographic waves over the continental slope, although we do not detect such a preference among the moorings examined. Further surveys using data from a global circulation model could be of value in this regard.

Others have addressed the vertical structure of ocean eddies. The construction of [Scott and Furnival \(2012\)](#) yielded improved results over standard flat bottom modes, but nevertheless required a phase-locked combination of two or more modes. The solutions of [Smith and Vanneste \(2013\)](#) account for non-zero density at the surface, but are also somewhat more involved in application. The present solution is perhaps the simplest of all possible solutions. And while it does not account for surface density, the results of [Wang et al. \(2013\)](#) and [LaCasce and Wang \(2015\)](#) suggest that it is not problematic below the surface mixed layer. If one requires predictions in the mixed layer, it makes sense to include an SQG component (assuming one knows the surface density).

As noted, a prevalence for the BT and BC1 mode is expected from idealized theory and turbulence simulations ([Fu and Flierl, 1980](#); [Scott and Arbic, 2007](#); [Smith and Vallis, 2001](#)) as well as from idealized ocean simulations ([Willebrand et al., 1980](#)). But these used a flat bottom geometry. Indeed, much of our fundamental theoretical understanding of the ocean is based on basins without

topography. The present results suggest it may be more fruitful to think in terms of topographic waves and rough bottom BC modes.

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